

Code Talker & Chomsky

- Who are “code talkers”?
- What would Chomsky say about them?
- Anything to do with time complexity?

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Ex C2

Part 1: MAX-CLIQUE

- $\{(G, k) \mid \text{the largest clique of } G \text{ has } k \text{ vertices}\}$
- Explain: “Unknown if it is in NP.”
- Speculate

Part 2: 3COLOR

- $\{G \mid \text{the nodes of } G = (V, E) \text{ can be distinctively colored with three colors}\}$
- $3\text{COLOR} \in \text{NP}$
- How to show $3\text{COLOR} \in \text{NPC?}$

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Mini Research Ideas

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Unit D3: Overview

- Compare time and space complexity
- Analyze the effect of nondeterminism with respect to space complexity
- Preview Exercise D3 “Mini Research Paper Outline”
 - Different types of exercises from this time on

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Time vs. Space

- They are not so different [paraphrase of Einstein].
- Time can be interpreted in another domain [applying Fourier transformation].
- One cannot be certain about both position (space) and momentum (time dimension involved ... velocity) [Heisenberg's Uncertainty Principle].
- Computers run faster with more memory [computer industry].

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Time vs. Space Complexity

- Time complexity
 - Maximum number of TM *steps* as a function of the input size (asymptotically)
- Space complexity
 - Maximum number of TM *tape space* as a function of the input size (asymptotically)

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Space Complexity Hierarchy

- Constant space
 - Any use?
- **LogSPACE** (also called **L**)
 - Handling inputs ... isn't it at least linear?
- LinearSPACE
 - SAT (an exhaustive algorithm) Focus in this unit
- **Polynomial space (PSPACE)**
 - Examples?



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$0^n 1^n$

- Review: Time complexity?
- Naive algorithm: LinSPACE
 - Usual use of a stack
- A LogSPACE algorithm possible?

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QBF Problem

- Quantified Boolean Formula
 - For all x , some y ($(x \text{ or } (\text{not } y))$ and $((\text{not } x) \text{ or } y)$)
 - " $x \text{ S } y ((x \cup \emptyset y) \cup (\emptyset x \cup y))$ Satisfiable?
 - Where each variable is either true or false
 - Cf. game-theoretic interpretation of FOL
- QBF Problem: $\{ \varphi \mid \varphi \text{ is a true Quantified Boolean formula} \}$
- Review: Time complexity?
- Space complexity?

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MAZE

- Finding a path between two points
 - Also known as graph-reachability problem
- Review: Time complexity?
- Space complexity?

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Nondeterministic Space

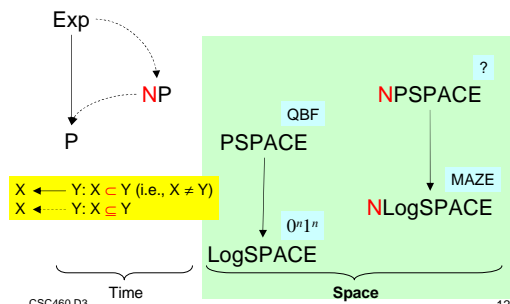
- Nondeterministic space complexity
 - Maximum number of **NTM tape space** as a function of the input size
 - Space measure for the successful run
- **NLogSPACE** (also called **NL**)
 - MAZE
- **NPSpace**
 - Example?



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Complexity Hierarchy



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Today's Questions

- X vs. $XSPACE$
 - E.g., P vs. $PSPACE$, NP vs. $NPSPACE$
- $XSPACE$ vs. $NXSPACE$
 - E.g., $PSPACE$ vs. $NPSPACE$, $LogSPACE$ vs. $NLogSPACE$
- X vs. $NLog(X)SPACE$
 - E.g., Exp vs. $NPSPACE$, P vs. $NLogSPACE$

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Group Exercise: Question 1

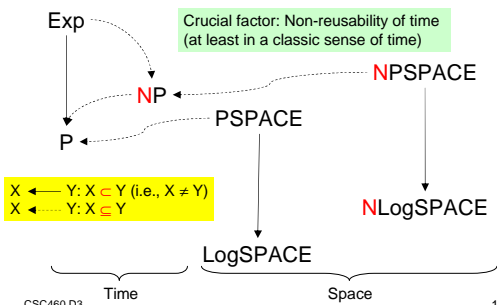
- X vs. $XSPACE$
 - E.g., P vs. $PSPACE$, NP vs. $NPSPACE$
- Which would be correct? Why?
 - $X \subset XSPACE$
 - $X \subseteq XSPACE$
 - $X = XSPACE$
 - $X \supseteq XSPACE$
 - $X \supset XSPACE$

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Complexity Hierarchy

Rev. 1



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Group Exercise: Question 2

- $XSPACE$ vs. $NXSPACE$
 - E.g., $PSPACE$ vs. $NPSPACE$, $LogSPACE$ vs. $NLogSPACE$
- Which would be correct? Why?
 - $XSPACE \subseteq NXSPACE$
 - $XSPACE = NXSPACE$
 - $XSPACE \supseteq NXSPACE$
 - None of the above

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Effects of Nondeterminism

Observation so far

- $L(TM) = L(NTM)$
- $L(DFA) = L(NFA)$
- $L(DPDA) \subseteq L(PDA)$ PDAs are nondeterministic
- $P \subseteq NP$
 - Probably $P \subset NP$ (i.e., $P \neq NP$)

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Group Exercise: Question 3

- X vs. $NLog(X)SPACE$
 - E.g., Exp vs. $(N)PSPACE$, P vs. $NLogSPACE$
- Which would be correct? Why?
 - $X \subseteq NLog(X)SPACE$
 - $X = NLog(X)SPACE$
 - $X \supseteq NLog(X)SPACE$
 - None of the above

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PSPACE-Complete

- Definition: PSPACE-complete
 1. The problem is in PSPACE.
 2. Every PSPACE problem is polynomial time reducible to the problem.
- Theorem: QBF is PSPACE-complete.
- Proof idea
 1. Existence of a PSPACE algorithm
 2. Simulate polynomial-time TM (with a technique similar to the proof of Savitch's Theorem)

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Unit Summary

- Time and space complexities have some connection.
- The effect of nondeterminism differ in each case.
- Applications of complexity theory
 - NPC and beyond \Rightarrow Use of approximation algorithms
 - Effect of limited resources (of different types)

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Summary Question

- What is your sense of time-space tradeoff, in the context of your choice (CS or else)? Discuss freely, referring to some example.
- Questions/Comments/Suggestions

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