# Unit C4: Graphs/Trees, 11/4/03

Exercise 1: Seating Arrangements

Due to the noisy situation at a banquet hall, we must observe certain conditions for all seating arrangements. These conditions are formally written as follows ("x 🗣 y" can be interpreted as "x can talk to y"):

Conditions:
1. $\forall x \, \neg(x$ 🗣 $x)$
2. $\forall x \forall y \, ((x$ 🗣 $y) \rightarrow (y$ 🗣 $x))$
3. $\forall x \forall y \forall z \, (((x$ 🗣 $y) \wedge (y$ 🗣 $z)) \rightarrow \neg(x$ 🗣 $z))$
4. $\forall x \exists y \exists z \, (y \neq z \wedge (x$ 🗣 $y) \wedge (x$ 🗣 $z))$

A. Compare **Condition** 3 with the formal definition of "transitive," and explain the difference.

B. The diagram below shows a seating arrangement using 2 tables. Each point indicates a person and the lines between two people indicate the possibility of talking to each other. Analyze whether this seating arrangement is consistent with the conditions. Discuss each statement in **Conditions**.
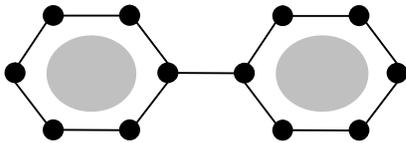
C. Find the smallest party that can satisfy all **Conditions**. Discuss each statement in **Conditions**.

D. Suppose that another party is considering a seating arrangement that can be represented as a "tree" (as defined in class). Would such an arrangement (as a structure) be able to satisfy **Conditions**? Explain, referring to all the relevant statements of **Conditions**.

Exercise 2: Model Your Own Phenomenon

First, identify a *unique* real-world phenomenon (as usual, you may simplify it to a level reasonable for this exercise) that can be represented by one of the following graph/tree types:
- Directed graph
- Undirected graph
- Weighted graph (directed or undirected)
- Tree

Then, formally define the structure with the following points in mind (as we practice in class):
- Give it a name.
- Define it as *n*-tuple of sets, relations, and/or functions.

- Formally define each structure component (with types where applicable). You may informally define sets of natural numbers, integers, and/or real numbers. You must define all other sets, relations, functions, formally.

Note that you must find a phenomenon that has not been discussed in class, exercises, sample mini projects, etc. Note that it is acceptable if independently working students come up with similar phenomena. However, if you work in a group, group members should not model a single phenomenon.