Unit B2: Relations, 9/26/03

Exercise 1: Professionals

Let us revisit the situation involving professionals. The conditions (logic) are as follows:

1. Everyone is mad.
2. There is at least one doctor.
3. There are at least two lawyers.
4. Doctors are not lawyers.
5. Lawyers sue everyone.
6. Doctors sue back if they are sued.
7. There is an individual who does not sue.

Recall the smallest structure (smallest number of people) that satisfies all of the above conditions, and use it for the following discussion.

A. Give formal definitions of the following structure components:

- Set of people (give arbitrary names to distinguish them), P
- Set of lawyers, L
- Set of doctors, D
- Set of mad people, M [try to give the most compact representation of this]
- Relation "sues", S

B. For every pair of sets discussed in Question A (i.e., P, L, D, and M, not including the relation S), give a formal representation to describe the relation between them (e.g., using $\in$, $\subseteq$, $=$, and their variations, as well as the use of set operations as a part of the representation). For example, can you say that $L \subseteq D$ and/or $L \cap M = \varnothing$?

C. Give the "type" (as introduced in class, but not in the textbook) of the relation S.

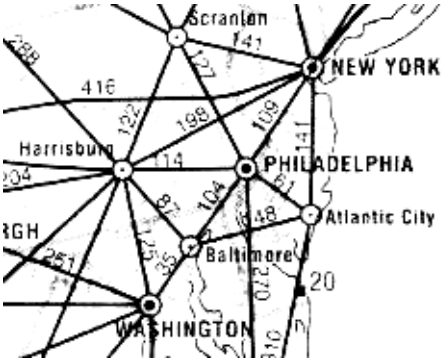
D. Formally represent that one person does *not* sue *anyone*, referring to the relation S.

Hint: The representation $(x, y) \in S$ means that x sues y. You must know how negative information is represented in the relation.

E. Would it be possible to consider the set M as a unary relation? Explain.

Exercise 2: Electronic Map

Consider the following cities shown on the map on right (use the abbreviation): Scranton (*s*), New York (*n*), Harrisburg (*h*), Philadelphia (*p*), Atlantic City (*a*), Baltimore (*b*), Washington D.C. (*w*). Also consider routes between cities (*only* those shown on the map and connecting the specified cities).



A. Give a formal representation of the map as a structure consisting of two components: (1) set of cities, *C*, and (2) routing relation (between two cities), *R*.

   Note: Recall that pairs are directional. Naturally, you must allow travel between two cities in both directions.

B. Is the relation *R* reflexive, irreflexive, or non-reflexive? Explain.

C. Is the relation *R* symmetric, antisymmetric, or non-symmetric? Explain.

D. Is the relation *R* transitive or non-transitive? Explain.

E. Create the *smallest* subset *S* of *R* so that travel from any city in *C* to any other city in *C* is possible by following the routes in *S*.

F. Is the relation *S* symmetric, antisymmetric, or non-symmetric? Explain.

G. Consider $T = R - S$. Would it be possible to travel from any city in *C* to any other city in *C* by following the routes in *T*? Explain.

Exercise 3: Pairs and *n*-Tuples as Set

Note: This is a slightly advanced question. A reasonable attempt will be considered as completion of this exercise.

A. Recall the definition of relation as a set of pairs/*n*-tuples. If pairs and *n*-tuples can be represented as a set, relation can be defined using sets, not referring to pairs/*n*-tuples. Either (1) come up with your own idea about how to do this or (2) find the way in the literature (including web search) and then justify that your representation satisfies the properties of relation (i.e., combines multiple objects and ordering is important).

B. [For this question, assume that you have an answer to Question A.] Discuss the consequence of the answer to Question A. Recall Unit B2 summary exercise.

<End>